Technical Note on Methods for Optimal Trajectory Synthesis

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In the existing racing world, race results, or even whole championships, are decided by details in racing packages - race car design specifications, race strategies, optimum car set-ups or drivers' capabilities to adapt to a certain track.

Two main approaches can be applied applied when searching for the weakest points in the racing package:

- Physical experiments (data acquisition systems, wind tunnel measurements);
- Computer aided engineering (e.g., FEM, CFD simulation and optimisation).

Since the computational power was increasing in last decades of the 20th century, engineers' effort was moving from mere experiments to a rational balance between computer aided engineering and the physical experiments. Computer model gives cost effective and easy to interpret results, but cannot reflect the reality completely. It can help to understand the real case, but only the real experiment can verify the results of computer modelling; and, vice versa, the theoretical computation is often used to verify the experimental data.

Hence an optimal trajectory synthesis is defined. The optimal trajectory synthesis (together with optimal velocity profile generation) is a method commonly used in professional racing. It helps to discover the latest tune-up possibilities of the racing package performance. Specifically, the optimal trajectory and the optimal velocity profile are tools to analyse the driver's ability of keeping the race car on its limit. A correlation between the optimal trajectory synthesis and the experimental data obtained from race car telemetry is able to disclose driver's style limits and create recommendations for improvement.

1 STUDY OF APPROACHES

The optimal trajectory synthesis task can be found in different fields - both academic and industrial - and in different branches, e.g. military, aerospace or in robotics. The current study of approaches is focused on dealing with optimal trajectory synthesis issues and necessary methods for vehicles, in both academic field and automotive industry.

1.1 Spline Trajectory and Genetic Algorithms

In Optimization of the Driving Line on a Race Track, Mühlmeier and Müller (2002) present the optimal trajectory synthesis for a Le Mans prototype race car. The vehicle is considered as a single mass point. The dynamic model is based on simple approaches in terms of vehicle traction, vehicle power and aerodynamics. The trajectory is determined using cubic spline curves. The lap time as a cost function is calculated integrating the equation of motion along the trajectory. The optimal velocity profile is based on the iterative determination of the brake points. The optimisation algorithm iteration step consists of changeable cubic spline controlling points P_i and uses the genetic algorithms to generate new population of drive lines. The resulting drive line is in Figure 1. Authors conclude that the optimal drive line significantly depends on the track and the vehicle properties.

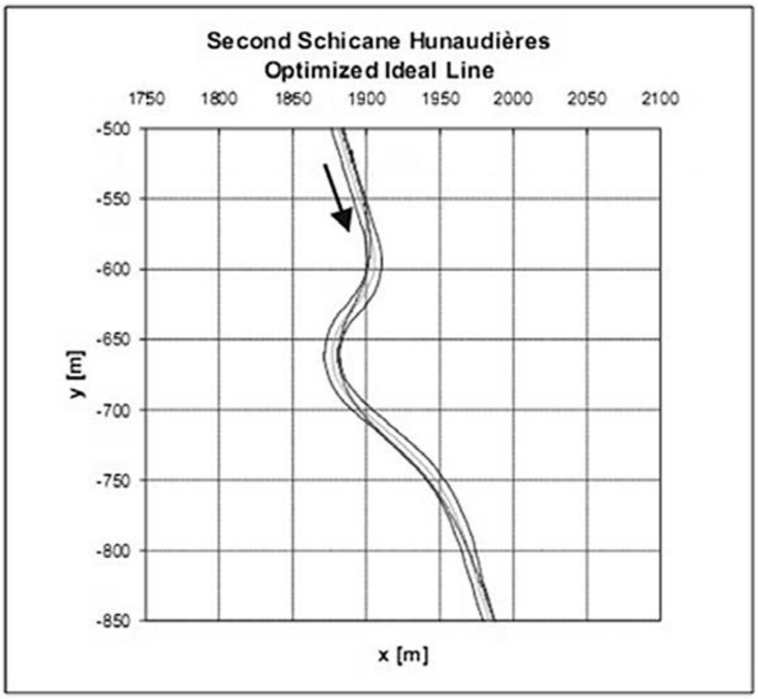

Figure 1: Optimized Ideal Line.

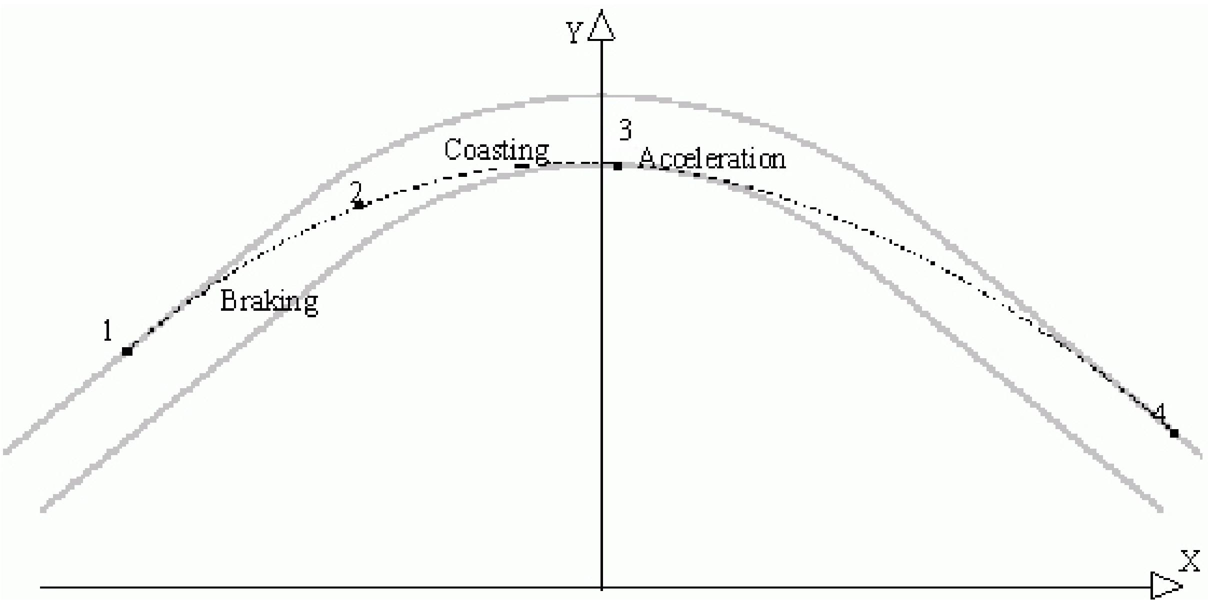

Figure 2: Spline curve trajectory control points (Cambiaghi et al., 1996).

The authors Cambiaghi, D., Gadola, M., Manzo, L., Vetturi, D. (1996) use a similar approach as before, together with the verification of a lap time simulation. The vehicle model is simplified and consists of a non-linear tyre model. The optimisation task includes the determination of 4 important points on the vehicle trajectory described by the cubic spline curve, see Figure 2: turn in point (1), brake release point (2), throttle application point (3) and corner exit (4). The genetic algorithms are used as the optimisation method again.

1.2 Semi-analytical Method for Velocity Profile Generation

In Profile Generation for Given Acceleration Limits, Velenis and Tsiotras (2005) propose a_t semi-analytical method for the optimal velocity profile generation on a given trajectory for a half-car model case. The vehicle is treated as a single mass point with given acceleration capacity, longitudinal force f^{max} and lateral force f_n^{max}, see Figure 3.

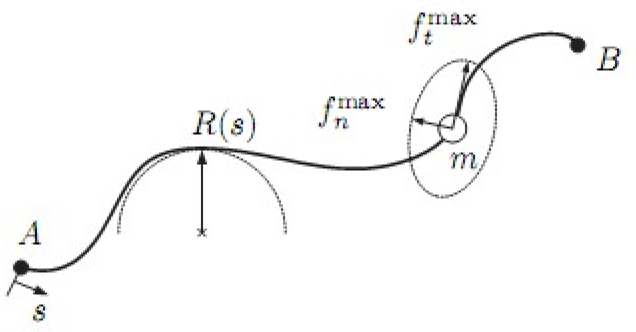

Figure 3: Trajectory for optimal velocity profile generation (Velenis & Tsiotras, 2005).

The optimal velocity profile is searched for on a curve described by partially linear radius function R= R (s), see left part of Figure 4. The search for optimal trajectory starts in finding local minima of the curve radius. In such points, critical velocity vcrit (maximum achievable velocity) is computed. Then the velocity characteristics composed of maximum available braking (before radius local minimum) and maximal available acceleration (after radius local minimum) are designed for each linear part of the curve radius. The minimum of these characteristics in each radius segment determines the optimal velocity profile, see the right part of Figure 4.

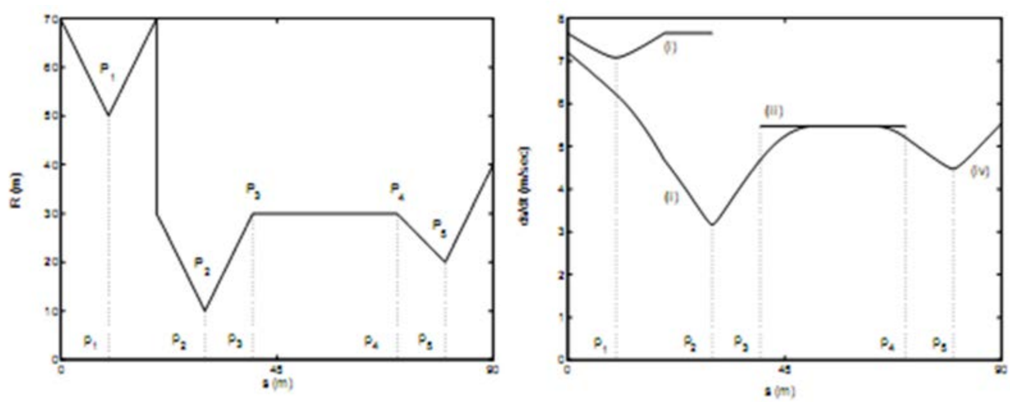

Figure 4: Curvature function (Velenis & Tsiotras, 2005).

2 CONCLUSION

The methods mentioned above show different attitudes to the optimal trajectory synthesis. The basic difference is in the definition: whether the problem is solved indirectly applying the optimisation techniques or directly using the optimal control.

The optimisation approaches evaluate the optimal velocity profile on a given track in each optimisation loop. The advantage of this approach is in the simple description of the mechanical model and therefore the generation of the optimal velocity profile is not exigent of computational power. On the other hand, the optimisation of the trajectory requires high computational power for searching in the set of the allowed curves. The approach described in “A Tool for Lap Time Simulation” is very close to the real driver behaviour when choosing the path and the velocity profile. The optimal control approach is rather different. It deals with the issue in a sophisticated way - it computes the velocity profile directly as a result of an applied optimal control theory approach. The optimal control allows to implement a more precise vehicle model.

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