Determination of Trajectory of Articulated Bus Turning along Curved Line

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ABSTRACT: The paper focuses on the presentation of a single-track dynamic model of articulated bus, a simplified linear model, which is based on reducing the double-track dynamic model with suitable assumptions that the front body lateral accelerations and rotational angles of the articulated bus are small. On the other hand, the phenomena of friction and interstice, such as articulated system (fifth wheel), are neglected. A coordinate system is defined to precisely describe the translational and rotational motions of the vehicle. In details, the author provides the dynamics model, sets up the kinematics and dynamic relations and builds differential equations of kinematic parameters presenting the articulated bus motions. These are objective functions, which are necessary to determine the gravity centre trajectory of the articulated bus, movement of the joint system and bodies of vehicle in addition to the swept path width of articulated bus. Matlab-Simulink is used to solve mathematical problems and simulate responding dynamics of vehicle. The modelling results are the functions of kinematic and dynamic parameters that allow to determine the motional trajectory and the swept path width of articulated bus, joint system and bodies of vehicle. These suitable results represent the fundaments for exact evaluation of the dynamic model and investigation of dynamics of articulated bus at a higher and more complex level.

KEYWORDS: Articulated bus, trajectory, dynamic model, articulated system.

1 INTRODUCTION

Nowadays, roads are becoming more and more congested, with increasing economic losses due to delays resulting from traffic jams and road accidents. Such problems may be reduced by using public transport systems. In order to make a public transport system attractive, it has to be cheap, fast and reliable. In cities, many types of public transport are in use, subways, trams and buses being the most common. However, each type has its advantages and disadvantages.

Articulated buses with their advantageous features in carrying capacity, transport productivity and reduction of transport costs have been exploited effectively. However, traffic safety of the above mentioned vehicle type should be improved and this topic is outlined in our specific study. Accidents which occur to buses usually cause serious consequences,

great damage on human lives and property. The mathematical study of the motion trajectory can contribute to the safety of articulated buses.

There are two basic conceptions of articulated buses:

- Articulated bus with engine in the front body (so-called puller articulated bus);
- Articulated bus with engine in the rear body (so-called pusher articulated bus, which is the main type and more popular nowadays.

The paper presents the motion of articulated buses on the road plane, which can be considered as a displacement of a double physical pendulum with two masses (front body and rear body of the vehicle).

2 THE MAIN OBJECTIVES OF RESEARCH

Since the aim of the research is to determine the motion trajectory and swept path width of articulated bus, the objective functions of this issue are given:

- Lateral (side) slip angles of front body and rear body of articulated bus: α_1 , α_2 and their derivatives: $\dot{\alpha}_1$; $\dot{\alpha}_2$; $\ddot{\alpha}_1$; $\ddot{\alpha}_2$;
- Yaw angles of front body and rear body: ε_1 , ε_2 and their derivatives: $\dot{\varepsilon}_1$; $\dot{\varepsilon}_2$; $\ddot{\varepsilon}_1$; $\ddot{\varepsilon}_2$;
- Rotational angle of front body to rear body: φ and its derivative: $\dot{\varphi}$; $\ddot{\varphi}$;
- Direction velocity vectors of front body and rear body: V₁; V₂.

3 DYNAMIC MODEL OF ARTICULATED BUS

The dynamic model of articulated bus with appropriate assumptions (Vlk, 2005; Nguyen Khac Trai, 1997):

- Articulated vehicle moves on a smooth and flat road;
- Tyre side slip angles of front steering wheel are the same;
- Front and rear bodies are absolutely rigid;
- Only the variability of kinematic and dynamic parameters on the ground plane are considered;
- The air resistance is neglected;
- The deformation of tyre is linear and the elastic moment is neglected;
- Joint is assumed as an ideal knuckle joint with no interstice and no frictional joint moment. The joint is located slightly ahead of the centre of the tractor rear axle.

3.1 Double track linear dynamic model

The motion of articulated bus is defined by different translational and rotational components. The linear velocity and acceleration of the vehicle, forces and moments are the motion phenomena of the vehicle during the ride. Two coordinate systems in the SAE convention (Society of Automotive Engineers) and ISO (International Standards Organization) are used for the vehicle dynamic simulation. The vehicle coordinate system has its origin at the centre of gravity of the vehicle. All movements of articulated vehicle bodies are given within the reference to this co-ordinate system.

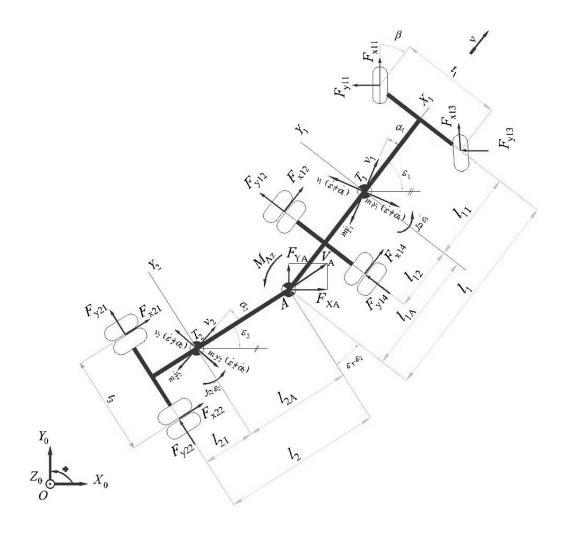


Figure 1: Double track model of articulated bus on planar phase.

The inertial (Earth fixed) coordinate system $(OX_0Y_0Z_0)$ is selected to coincide with the vehicle coordinate systems $(T_1X_1Y_1Z_1)$ on the front body and $(T_2X_2Y_2Z_2)$ on the rear body. The absolute motion quantities are defined with respect to this coordinate system and the components of these quantities are defined along the axes of the vehicle fixed coordinate systems. The orbit motion of articulated bus is determined by the fixed coordinate system $OX_0Y_0Z_0$.

With this coordinate system, it can be assumed that articulated bus is a mechanical system with two solid bodies. The first one has the centre of mass at the centre of the front body and the other one at the centre of the rear body. The knuckle joint system (point A) connects the two bodies.

On the $T_1X_1Y_1Z_1$ axis system: the origin of the axis system is the centre of front body (T_1) . The X_1 axis is the front body longitudinal motion. The Y_1 axis is in lateral direction and in the ground plane. The Z_1 axis is perpendicular to the ground plane with a positive direction upward. The positive direction of rotational angle is counterclockwise.

On the $T_2X_2Y_2Z_2$ axis system: the centre of gravity co-ordinate system, whose origin is the centre of the rear body (T_2) . The X_2 axis is the rear body longitudinal motion. The Y_2 axis is lateral direction and perpendicular to X_2 axis. The Z_2 axis is perpendicular to the ground plane. The positive direction of the rotational angle is counterclockwise.

3.2 Single – track linear dynamic model

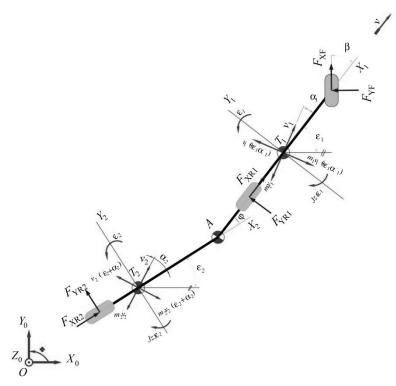


Figure 2: Single track model of articulated bus on planar phase.

The overall dynamic model of articulated bus is shown on respective figures. In order to build up vehicle motion equations, dynamic functions were determined and the connecting joint of the front body of articulated bus was separated from the rear body and replaced by constraint reactions.

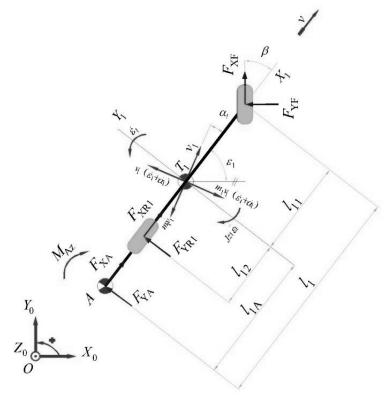


Figure 3: Single - track model on front body of articulated bus behind separating joint.

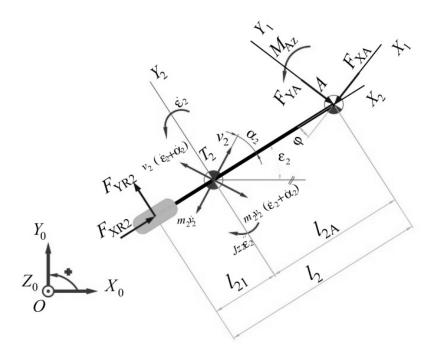


Figure 4: Single - track model on rear body of articulated bus behind separating joint.

The equations of the motion with respect to the axles fixed to the front or rear body are given and after reducing these equations, four differential equations are shown:

$$\begin{split} \dot{v}_{1} &= \frac{1}{m_{1}} \left((F_{XF} \cos \beta - F_{YF} \sin \beta + F_{XR1} + F_{XA}) \cos \alpha_{1} + \right) \\ &+ (F_{XF} \sin \beta + F_{YF} \cos \beta + F_{YR1} + F_{YA}) \sin \alpha_{1} \\ \dot{\alpha}_{1} &= \frac{1}{v_{1}.m_{1}} \left(-(F_{XF} \cos \beta - F_{XF} \sin \beta + F_{XR1} + F_{XA}) \sin \alpha_{1} + \right) - \dot{\varepsilon}_{1} \\ \ddot{\varepsilon}_{1} &= \frac{1}{J_{Z1}} \left(\left[F_{XF} \sin \beta + F_{YF} \cos \beta \right] J_{11} - F_{YR1} J_{12} - F_{YA} J_{1A} \right) \\ \ddot{\varphi} &= \frac{1}{J_{Z2}} \left[F_{YR2} J_{21} + (F_{YA} . \cos \varphi + F_{XA} \sin \varphi) J_{2A} \right] + \ddot{\varepsilon}_{1} \end{split}$$

3.3 Trajectory and swept path width of articulated bus

During the motional process, the tractor gravity center changes position and it is determined from instantaneous velocity V_1 making tangent line to curve of orbit considered in the fixed intertial system.

The motional trajectory of articulated bus is defined by the 3 following points: the centre of gravity of tractor, the joint and the centre of gravity of semi-trailer (Dang Hoang Anh, 2009).

The centre of gravity location of the front body of articulated bus at T_1 :

$$X_{T1} = \Delta X_{10} = \int_{t_i}^{t_i + \Delta t} V_{1Xo} dt = \int_{t_i}^{t_i + \Delta t} V_1 \cos(\alpha_1 + \varepsilon_1) dt$$

$$Y_{T1} = \Delta Y_{10} = \int_{t_i}^{t_i + \Delta t} V_{1Yo} dt = \int_{t_i}^{t_i + \Delta t} V_1 \sin(\alpha_1 + \varepsilon_1) dt$$

Joint location A:

$$X_A = X_{T1} - l_{1A} \cos(\varepsilon_1)$$

$$Y_A = Y_{T1} - l_{1A} \sin(\varepsilon_1)$$

The centre of gravity location of the rear body of articulated bus at T₂:

$$X_{T2} = X_A - l_{2A}\cos(\varepsilon_1 - \phi)$$

$$Y_{T2} = Y_A - l_{2A}\sin(\varepsilon_1 - \phi)$$

The swept path width of articulated bus in motion is determined by the location of the outer edge point in proportion to limited dimensions of the vehicle.

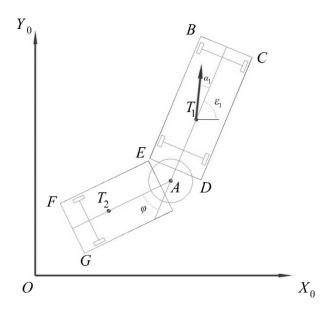


Figure 5: Swept path width of articulated bus with limited dimensions.

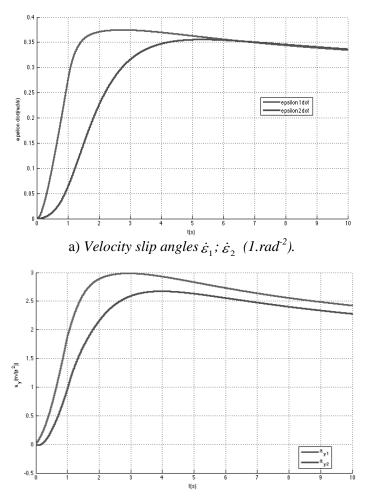
4 SIMULATION

Excitation functions of the mechanical system of articulated bus occur in the above mentioned equations.

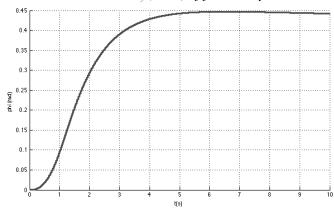
- Excitation function of the turning angle of the steering wheel: Excitation function of the steering wheel is specified by the front wheel turn angle of the vehicle front body. The rule of the front wheel turn is shown in the figure below; in stable wheel turning conditions the front wheel will turn at the maximum angle of 0.14rad (8°);
- Longitudinal force function: In the content of the research, we suppose that articulated bus moves in a stable way and the total tractive force produced by the driven wheel equals the total rolling resistance of wheels in vehicle axles.

5 SIMULATION RESULTS

The Matlab – Simulink tool was used for vehicle dynamics simulation. The total time for the program simulation is 10 seconds. The velocity of the vehicle is constant 8m/s. The results are shown on the figures below:



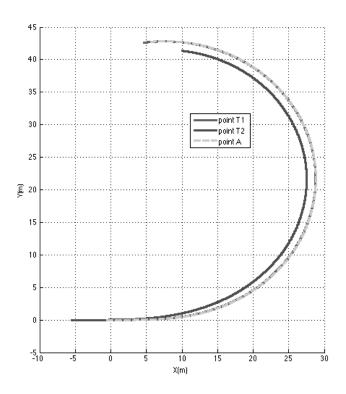
b) Acceleration slide side a_y (m.s⁻²) of front body and rear body.



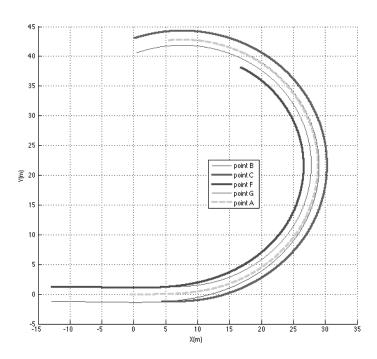
c) Slip angles of front body with rear body φ (rad).

Figure 6: Simulation dynamic of articulated bus turning on curve line.

The dynamics of articulated bus in stable conditions when turning on curved line without break forces (this is a basic case in comparison with another cases) shown in the figure above. The figures present the response of vehicle dynamics.



a) Trajectory of centre gravity points.



b) Swept path of vehicle by trajectory of points.

Figure 7: Determination of swept path hand trajectory of centre gravity points of vehicle.

Figure 7 presents points trajectory: the centre of gravity T_1 of the front body with the starting point has co-ordinate (0,0); Joint A with the starting point has co-ordinate (-0.5,0); the centre of gravity T_2 of the rear body starting point has co-ordinate (-5.5,0).

- During the first initial second of translational movement, (in the first 8 meters), the front body of articulated bus moves mainly forward;
- Translational movement is finished to maintain the steering wheel angle, trajectory of articulated bus continues to change and follows a stable circle.

7 CONCLUSION

Within its limited content, the article presents the trajectory of tractor-semitrailer on a single-track linear model with appropriate assumptions.

However, this was examined in an area of minor changes with little lateral acceleration. Research in the areas of major changes (non-linear area) is necessary for the creation of a complex model.

The research results can be used as reference material for following dynamic research with a complete spatial model.

At the same time, the method of calculation as well as the vehicle design parameters indicate a driving corridor at the road plane.

Besides, it is possible to explore situations which influence the structural data in order to optimize the movement of the vehicle.

ACKNOWLEDGEMENTS

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